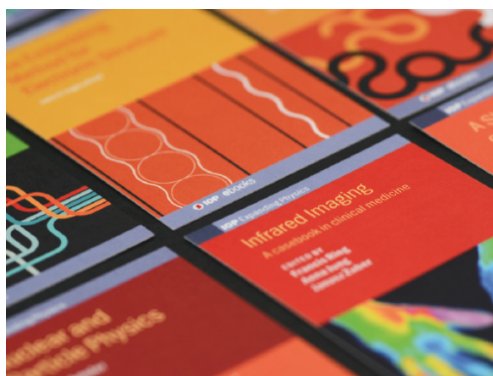


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# Why is Mathematical Representation Difficult for Students?

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**Abstract.** Mathematical representation is one of the mathematical skills that can help students present mathematical problems in various forms. However, students often face difficulties in doing so. This study aims to investigate the causes of difficulties faced by students in solving mathematical representation questions on trigonometry of the 10<sup>th</sup> grade. This is a qualitative case study. Data were collected both by documentation of research results in the form of 6 essay questions on mathematical representation and by interviews. The research participants were 10 high school students in the 10<sup>th</sup> grade of science program in Cirebon Regency, West Java, Indonesia. Through this case study, found the causes of difficulties experienced by students in solving mathematical representation questions. Most difficulties (10 students) occur because students cannot represent mathematical expressions/symbol to solve questions in the form of images. On the other hand, the least difficulties (6 students) occur because students cannot use image representation to solve written text questions.

## 1. Introduction

Mathematical skills that students should learn include: (1) problem solving, (2) reasoning and proof, (3) communication, (4) connections, and (5) representation [1]. In line with that, Callison revealed that students need to master mathematical skills, such as solving problems, making connections, understanding various representations of mathematical ideas, communicating ideas, and explaining reasoning [2]. Mathematical representation is one of the important mathematical skills that students need to master because it serves as the foundation for students in order to comprehend and utilize mathematical ideas. Therefore, it is necessary to investigate if students find mathematical representation challenging [3]. NCTM argued that mathematical representation is the most important and the most needed skill to be developed as mathematical representation functions as the basic skill in learning mathematics [4].

Representation refers to the results of students' interpretation of a problem used to help find solutions [5]. Aspects of mathematical representation include pictures, graphic, tables, equations, mathematical symbols, words, and writings. Due to a lot of aspects of mathematical representation, students need to repeatedly learn how to solve problems containing those aspects with the purpose of developing their ability to organize ideas when solving the problems. Students can use representation to find and create a way of thinking in communicating their mathematical ideas from an abstract concept into a concrete one that enables them to understand the problems more easily [6].



According to NCTM, one of the students' weaknesses in learning mathematics is that they are not able to connect mathematical concepts with daily experiences. Learning mathematics is often viewed to be too formal and not to really associate with meaning, understanding, and application of mathematical concepts [7]. These perceptions signify that problems occur when students attempt to solve applicative essay questions, especially questions about mathematical representation. To find out problems and difficulties that students encounter, students are given several questions that tested students' mathematical representation skill. The test is a set of instruments used to take measurements [8]. The measurements in this regard refer to the measurements of mathematical representation skill to identify difficulties that students face in solving mathematical representation questions. If no actions are taken to tackle these difficulties, it is possible that students will experience more difficulties in other areas of mathematics and mathematical skills. Some studies mention that students face mathematical difficulties because they often miscalculate [9], are unable to comprehend mathematical symbols [10], lack counting skills [11], and lack skills to understand mathematical language [12].

The appropriate way to investigate students' difficulties is to examine students' answers to the questions in a test. After the students' difficulties are successfully examined, the results can be used by teachers to review the points of difficulties. The study by Fauzan that aims to discover the points of students' difficulties in high school mathematics subject found that students were detected to have difficulties in geometry as they considered it the most difficult area of mathematics [13]. The difficulties were specifically related to the concepts and procedures in solving mathematical questions.

The results of interview with 10 students in the 10<sup>th</sup> grade indicated that besides geometry, another difficult area of mathematics for them was trigonometry. Trigonometry is a lesson discussed in compulsory mathematics for the 10<sup>th</sup> grade students. The topics include trigonometric ratios in various quadrants, trigonometric ratios in related angles, laws of sines and cosines, and trigonometric functions. Based on the interview results, the most difficult topic was trigonometric functions, although some students argued that all topics of trigonometry were difficult to be learned. Based on the explanation above, this study aims to examine why students experience difficulties in solving essay questions about mathematical representation. Furthermore, this study also investigates further the types of difficulties that students face in solving essay questions about mathematical representation.

## 2. Methods

A case study uses a qualitative approach in order to obtain information about a case thoroughly and deeply [14]. The research participants were 10 high school students in 10<sup>th</sup> grade in Cirebon Regency, West Java. The participants were selected based on the criteria according to the study [15], then taking the subject using a purposive sampling technique. The criteria were the 10<sup>th</sup> grade students who had already learned trigonometry and experienced difficulties in solving mathematical representation questions, specifically about trigonometry. Data were collected by observation of the results of

**Table 1.** Indicators of Mathematical Representation Skill.

No	Indicators of Representation
1	Use pictures in solving expressions/mathematical symbols (Mathematical Expressions/ Symbols → Pictures)
2	Use pictures in solving questions in the form of written texts (Written Texts of Mathematics → Pictures)
3	Use expressions/mathematical symbols in solving questions in the form of pictures (Pictures → Mathematical Expressions/ Symbols)
4	Explain expressions/mathematical symbols in solving questions in the form of expressions/mathematical symbols (Mathematical Expressions/ Symbols → Mathematical Expressions/ Symbols)
5	Use expressions/mathematical symbols in solving questions in the form of mathematical texts (Written Texts of Mathematics → Mathematical Expressions/ Symbols)

students' tests on mathematical representation and interviews. Data were later analyzed using Miles & Huberman theory that consists of three stages which are data reduction, data display, and conclusion drawing [15]. The following are indicators of mathematical representation ability used to identify difficulties experienced by students in working on mathematical representation problems.

### 3. Results

Students' difficulties were observed and measured by using the indicators of mathematical representation skill on each question given to 10 students of science program in 10<sup>th</sup> grade. The questions were developed by the researchers. Table 2 below illustrates the numbers of difficulties experienced by 10 students in solving mathematical representation questions.

**Table 2.** Numbers of 10 Students' Difficulties in Mathematical Representation.

Number	Indicators of Competence	Indicators of Representation	Difficulties
1A	Illustrate the values of trigonometric ratios in quadrant III	Mathematical Expressions/ Symbols → Pictures	7 students
1B	Determine the values of cosecant and cotangent ratios if the value of secant ratios is known in quadrant III	Mathematical Expressions/ Symbols → Mathematical Expressions/ Symbols	7 students
2	Determine law of cosines if two sides and an angle of a triangle are known	Written Texts of Mathematics → Mathematical Expressions/ Symbols	8 students
3	Determine the values of trigonometric functions using a unit circle with a $>360^\circ$ angle	Mathematical Expressions/ Symbols → Mathematical Expressions/ Symbols	7 students
4	Illustrate contextual problems of trigonometric ratios	Written Texts of Mathematics → Pictures	6 students
5	Analyze the changing graphs of trigonometric functions as a result of changes in constants in the function of $y = a \sin nx$	Pictures → Mathematical Expressions/ Symbols	10 students

Table 2 shows that the question number 5 with the indicator of mathematical representation "using mathematical expressions/symbols in solving problems in the form of pictures" is the most difficult question for students because all students experience difficulties. On the other hand, the least difficult question about mathematical representation for students is the question number 4. In question number 4, a student faced a difficulty in using mathematical expressions/symbols. The following is the explanation of students' difficulties in solving mathematical representation questions based on the indicators of mathematical representation employed in this study.

#### 3.1. Using pictures to solve mathematical expressions/symbols

The difficulties in the indicator of the first representation are presented in Figure 1(a) and 1(b) for the question number 1A. The indicator of competence is to determine the values of cosecant and cotangent ratios if the value of secant ratio in quadrant III is known. The instruction of the question number 1A is the value of  $\sec \alpha = -25/7$  and  $\pi < \alpha < 3/2\pi$ . Based on the mathematical expressions/symbols in the question, students were asked to illustrate the comparative value of secant  $\alpha$  in the Cartesian plane. However, students were not able to write mathematical expressions/symbols as instructed by the question. This difficulty happened to 7 out of 10 students. Therefore, when students are instructed to illustrate the value of trigonometry comparisons of the information contained in the question, 7 students experienced difficulties in drawing it as seen in Figure 1(b).

Figure 1(a) shows students' works indicating that they faced difficulties in writing mathematical expressions/symbols. In this regard, students made mistakes or were careless in writing mathematical expressions/symbols. Another difficulty, students find it difficult to understand the problem of mathematical expressions/ symbols is that students did not write any mathematical symbols for the instruction "known". This occurred because students are not accustomed to do essay questions in a structured and organized manner in which students are required to write the complete answer based on what is known in and asked by the question. According to previous studies, students faced difficulties because they lacked comprehension of the basic concepts of trigonometry and considered the materials difficult [16].

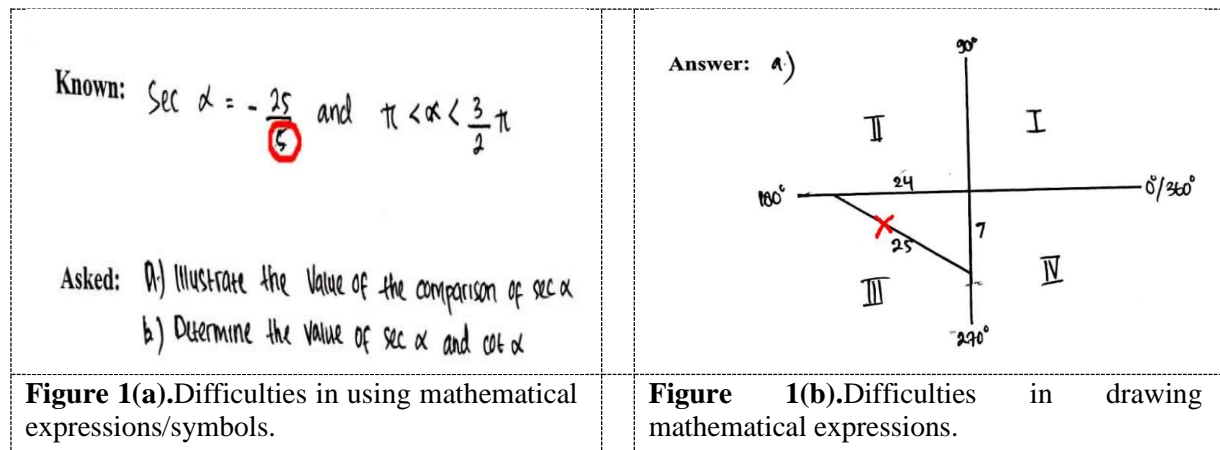
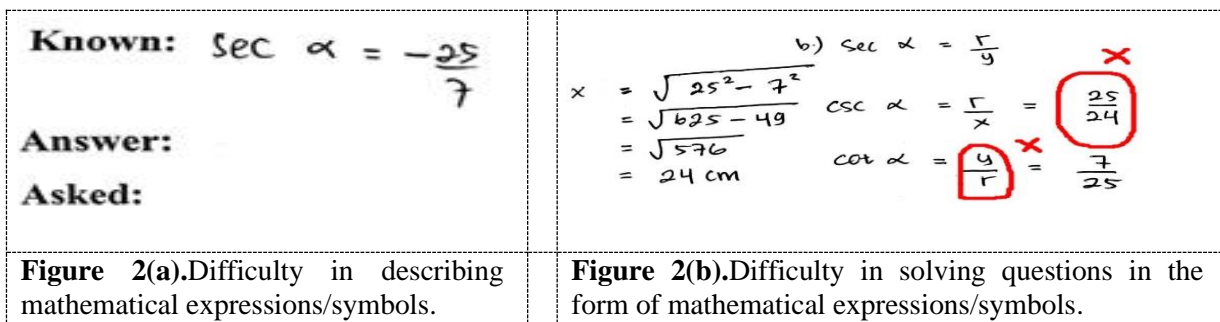
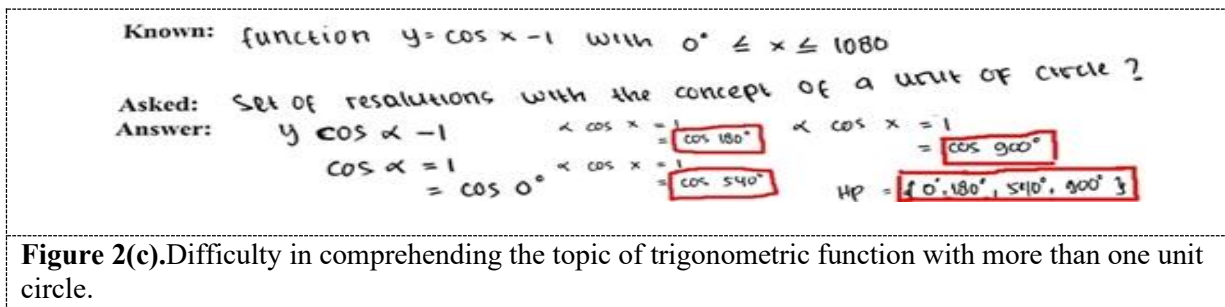


Figure 1(b) shows students' works in drawing mathematical expressions/symbols based on what is known in the question. In that figure, students faced a difficulty in drawing the comparative value of secant  $\alpha$ . The figure made by students is not seen in accordance with the concept of trigonometry. As a result of this difficulty, students cannot use representation rules for the next questions related to this question [17]. Based on the results of the study by Usman, students should be given adequate opportunity to learn how to do questions in a structured manner with the purpose of helping them improve their mathematical representation skill [18].

3.2 Describing mathematical expressions/symbols in solving questions in the form of mathematical expressions / symbols

Based on the second indicator, the point of difficulty is in the question number 1B. The indicator of trigonometric competence in the question number 1B is to determine the values of cosecant and cotangent ratios if the value of secant ratio in quadrant III is known. The instruction of the question number 1B is "determine the values of cosecant  $\alpha$  and cotangent  $\alpha$  if  $\sec \alpha = -25/7$  and  $\pi < \alpha < 3/2\pi$ ". Before determining the values of cosecant  $\alpha$  and cotangent  $\alpha$ , students wrote the information of expressions/symbols in the "known" part. However, 7 students did not write complete information as seen in Figure 2(a).





**Figure 2(c).** Difficulty in comprehending the topic of trigonometric function with more than one unit circle.

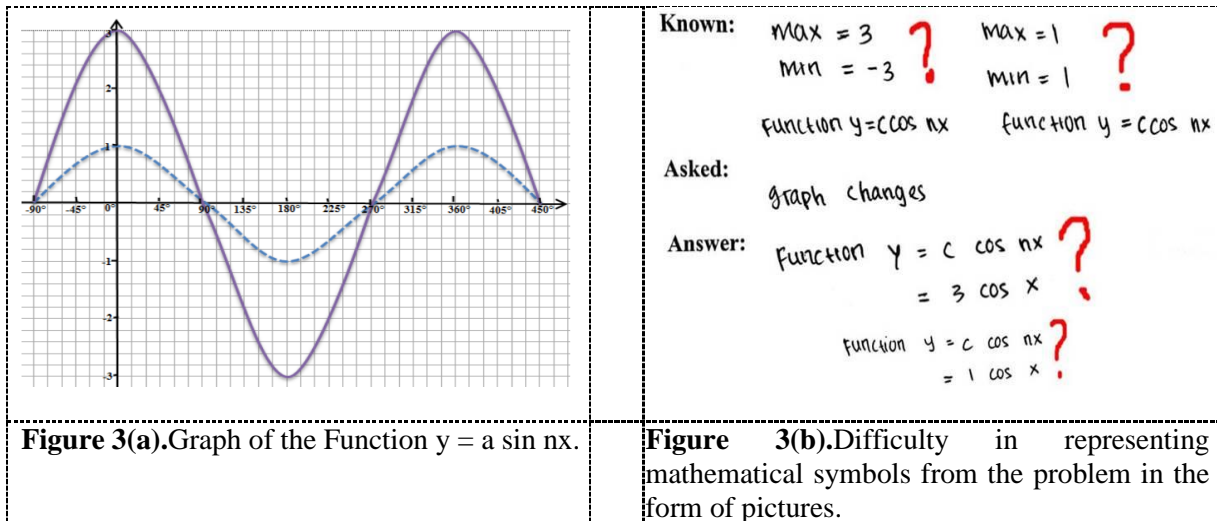
Figure 2(a) shows that students faced difficulties in writing mathematical expressions/symbols. Thus, they wrote only some parts of the expressions/symbols. Besides, to calculate the values of cosecant  $\alpha$  and cotangent  $\alpha$ , students first determine the length of an unknown side of the triangle based on the information given in the question. The unknown side is the front leg because it is known that the value of  $\sec \alpha$  is  $-25/7$  in quadrant III. The comparative secant value is the hypotenuse and the side leg; thus, the length of the front leg needs to be found. After the length of the front leg is found, the values of all three sides are used to determine the values of cosecant  $\alpha$  and cotangent  $\alpha$ . Students' difficulty in solving the question number 1B is presented in Figure 2(b). It can be seen that students have low comprehension of the basic concepts of trigonometry and the signs of  $+/-$  in each quadrant so that they obtained incorrect comparative value of cotangent  $\alpha$ .

The question number 3 contains the same indicator with the question number 1B. The difficulties that occur in questions number 3 and 4 are the same problem can give rise to errors from different sources, and the same error can arise from different problem-solving processes [19]. There were 7 students who experienced difficulties in this question. The question number 4 gives information of the function  $y = \cos x - 1$  with  $0^\circ \leq x \leq 1080^\circ$ . Furthermore, students are asked to determine the set of functions using the unit circle concept. The difficulty encountered by students is illustrated in Figure 2(c). It can be seen that students faced difficulties in comprehending the topic of trigonometric function with more than one unit circle. When looking for the first set of functions, students gave the right answer that  $\cos x = 1$ ; for  $x = 0^\circ$ . However, students' further answers were incorrect; the value of  $\cos 180^\circ$  is  $-1$ , but it is supposed to be  $-1$ . The same case happened to other answers. Therefore, the set of resolutions of the question number 3 is  $\{0^\circ, 360^\circ, 720^\circ, 1080^\circ\}$ .

### 3.3 Using mathematical symbols in solving problems in the form of pictures

The question number 5 contains the indicator of the third representation. The indicator of competence is to analyze the changes of the graph of trigonometric function as a result of the change of constant in the function  $y = a \sin nx$ . The question number 5 shows the graph changes so that the two graphs of functions can be determined. The following Figure 3(a) is the graph used in the question number 5.

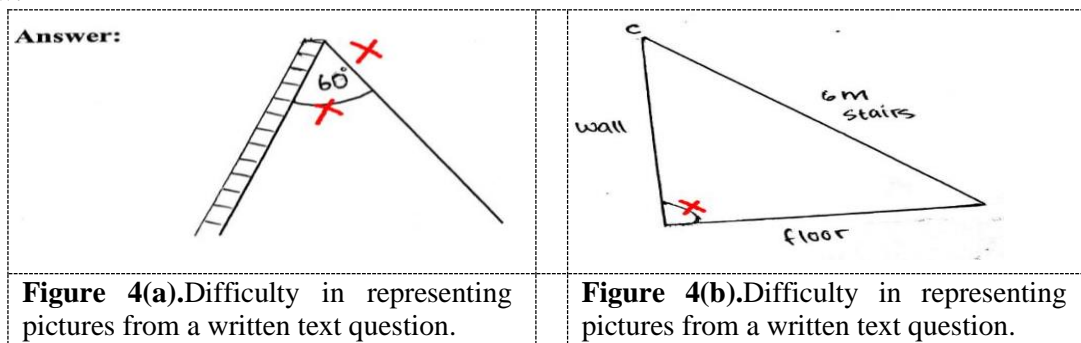
The difficulties most students faced occurred to the question number 5 in which 10 students did not write anything on the answer sheet. Students understood the graph, but they were not able to represent mathematical expressions/symbols given in the graph. One of the difficulties is presented in Figure 3(b) below.



Out of 10 students, all had difficulty in solving the question number 5 shown in Figure 3(b). Only a small number of students did as in Figure 3(b) and the rest did not answer. In this regard, students were not able to mention the reason of graph changes, even though they were able to read what the graph means and to understand the changes of both functions. However, they were not able to explain the reason or give evidence that the functions are  $y = \cos x$  and  $y = 3 \cos x$ . One of the reasons to prove that the function is  $y = \cos x$  is if  $x = 0^\circ$ , then  $f(x) = \cos 0^\circ = 1$  and if  $x = 90^\circ$ , then  $f(x) = \cos 90^\circ = 0$ . On the other hand, one of the reasons to prove that the function  $y = 3 \cos x$  is if  $x = 0^\circ$ , then  $f(x) = 3 \cos 0^\circ = 3 \times 1 = 3$  and if  $x = 90^\circ$ , then  $f(x) = 3 \cos 90^\circ = 3 \times 0 = 0$ . These two reasons are already in accordance with the graph that has dotted lines.

3.4 Using pictures as representation in solving questions in the form of written texts

The indicator of the fourth representation is contained in the question number 4 with the indicator of competence “to illustrate the contextual problem of trigonometric ratios of a right triangle”. There were 6 students who faced difficulties in solving this question. The question reads “Father leaned a ladder right at the top of the wall. The length of the ladder is 6 m, while the angle between the floor and the bottom end of the stairs is  $60^\circ$ . Illustrate this information with a picture.” Out of 6 students who faced difficulties, two of them illustrated the information as presented in Figure 4(a) and 4(b) below.



In Figure 4(a) and 4(b), students faced the same difficulty that is to represent pictures in order to solve a question in the form of written texts. The concept in this question relates to the concept of a triangle. students' difficulties in this concept are in accordance with the students' responses from Akhter research that, introduction to coordinate geometry and Parallelograms and triangles is difficult for students [20]. Figure 4(a) shows that it was difficult for students to locate the size of the known angle. The question tells that the angle of  $60^\circ$  is located between the bottom end of the ladder and the

floor, but students located the angle of  $60^\circ$  between the top of the ladder and the wall. On the other hand, based on Figure 4(b), students represented only the length of the ladder. They found it difficult to represent the size of the known angle. Both cases occurred because students are unfamiliar with questions in the form of written texts or with essay questions represented in pictures.

### 3.5 Using mathematical expressions/symbols in solving questions in the form of mathematical texts

The question number 2 contains the indicator of representation of using mathematical expressions/symbols in order to solve questions in the form of mathematical texts. The indicator of its competence is to determine law of cosines if two sides and an angle are known. There were 8 students faced difficulties in regard to this indicator. The question number 3 reads "An ABC triangle has sides a, b, and c. The triangle applies the formula:  $a^2 = b^2 + c^2 - (\sqrt{2})(bc)$  and  $c^2 = a^2 + b^2 - (\sqrt{3})(ab)$ , then determine the size of angle B of the triangle." A student solved the question as presented in Figure 5, but the other 7 students did not write anything or faced difficulties in solving this question.

In one of the students' answers presented in Figure 5, the student used the right concept, but after the value of  $\cos A = \frac{\sqrt{2}}{2}$  was obtained, he was not able to finish it until the end that is to find the size of angle A. In an attempt to find the size of angle B, students only did the question until they obtained the value of  $\cos C = \frac{\sqrt{3}}{2}$ , but they could not find the size of angle C. After the sizes of angle A and C were obtained, students were supposed to be able to find the size of angle B by subtracting the total of three angles of a triangle from the total of angles A and C. When doing the question number 2, students were not able to use mathematical expressions/symbols in order to solve questions in the form of mathematical text/essay.

Known:  $a^2 = b^2 + c^2 - \sqrt{2} bc$   
 $c^2 = a^2 + b^2 - \sqrt{3} ab$

Asked: Angles B ?

Answer:  $a^2 = b^2 + c^2 - \sqrt{2} bc$   
 $b^2 + c^2 - 2bc \cos A = b^2 + c^2 - \sqrt{2} bc$   
 $2bc \cos A = \sqrt{2} bc$   
 $\cos A = \frac{\sqrt{2} bc}{2bc}$   
 $\cos A = \frac{\sqrt{2}}{2}$

$a^2 + b^2 - 2ab \cos C = a^2 + b^2 - \sqrt{3} ab$   
 $2ab \cos C = \sqrt{3} ab$   
 $\cos C = \frac{\sqrt{3} ab}{2ab}$   
 $\cos C = \frac{\sqrt{3}}{2}$

**Figure 5.** Difficulty in representing mathematical expressions in solving a question in the form of written texts.

## 4. Discussion

Based on the results, in general, students still feel that mathematical representation skill is difficult. All five indicators of representation are considered difficult by students. Given the indicator of using pictures to represent mathematical symbols, students did not write complete and proper mathematical symbols. This means students faced difficulties in using pictures to represent symbols. As a result, their pictures were not in accordance with the concept of trigonometry. Another factor is that students experienced misunderstanding of the theory and did not comprehend mathematical symbols [10]. Furthermore, students also encountered difficulties in writing symbols from the comparison concept of trigonometry. This resulted in errors in calculation [12] and in using representation of the symbols (+) and (-) in regards to the comparison concept of trigonometry in each quadrant. The third indicator is using mathematical symbols to represent pictures. In this indicator, it was difficult for students to read and comprehend the pictures of graph changes of trigonometric function which caused students to unable to use mathematical symbols to represent pictures [12]. The difficulty in using pictures to represent written text questions happened to students when doing essay questions or questions related



to daily life. The last indicator is to use mathematical symbols to represent written text questions. In this indicator, students felt difficult to comprehend written text questions so that they were not able to use mathematical symbols to represent written text questions. Other students faced difficulties in certain stages; thus, they could not solve the problem until the final process [10]. This occurred because students lacked skills in calculation and comprehension of mathematical symbols.

The types of difficulties found in this study are (1) difficulty in using pictures to represent mathematical symbols, (2) difficulty in using mathematical symbols to represent mathematical symbols, (3) difficulty in using mathematical symbols to represent pictures, (4) difficulty in using pictures to represent written text questions, and (5) difficulty in using mathematical symbols to represent written text questions. Besides, students consider trigonometry as one of the materials in mathematics that is difficult and challenging to be learned. Given the difficulties found in this study, the information in this regard is expected to be useful for teachers to discover the proper ways to overcome difficulties experienced by students [10], because the ability of representation has a very important role in the learning of mathematics that need to be owned by each student.

## 5. Conclusion

Based on the results and the discussion above, it can be concluded that students experienced difficulties in solving mathematical representation questions. Students' difficulties were apparent from students' works in solving the questions given by the researchers. In addition, the results of interviews also suggest that it was difficult for them to do questions about mathematical representation. The questions designed and given to students were considered more difficult to be solved than the questions students usually received in the learning process. This happens because students lack practice in doing mathematical skill-based questions.

All indicators of representation skill contained in each question were considered difficult by students. Mathematical representation is difficult for students because of five reasons, including because students have difficulty in representing mathematical expressions/symbols in the form of images (7 students), students have difficulty in representing questions in the form of mathematical expressions/symbols in the form of expressions/symbols mathematics (7 students), students have difficulty in representing written text questions in the form of mathematical expressions/symbols (8 students), students have difficulty representing mathematical written text in the form of images (6 students), and students have difficulty in representing questions in the form of images in the form of mathematical expressions/symbols (10 students). After finding the cause of the difficulties felt by students in solving mathematical representation questions, it can be using as a teacher as a remedial or stabilization relating to trigonometric mathematical representation.

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